Ableitung der Umkehrfunktion von Areafunktionen.

Gegeben: f(x) = y = sinh(x)
Gesucht: arcsinh(x)

$$f(x) = y = \frac{1}{2}(e^x - e^{-x})$$

$$f'(x) = y` = \frac{1}{2}(e^x + e^{-x})$$

$$\cos^2 h(x) - \sin^2(x) = 1$$

$$\Rightarrow \cosh(x) = \sqrt{\sin^2 h(x) + 1}$$

$$\mathrm{g'(y)} = \frac{1}{f'(x)} = \frac{1}{\frac{1}{2}(e^x + e^{-x})} = \frac{1}{\cosh(x)} = \frac{1}{\sqrt{\sin^2 h(x) + 1}} = \frac{1}{\sqrt{y^2 + 1}}$$

Nun Variablen tauschen

$$y = \frac{1}{\sqrt{x^2 + 1)}}$$

Gegeben: f(x) = y = cosh(x)
Gesucht: arccosh(x)

$$f(x) = y = \frac{1}{2}(e^x + e^{-x})$$

$$f'(x) = y` = \frac{1}{2}(e^x - e^{-x})$$

$$\cos^2 h(x) - \sin^2(x) = 1$$

$$\Rightarrow \sinh(x) = \sqrt{\cos^2 h(x) - 1}$$

$$\mathrm{g'(y)} = \frac{1}{f'(x)} = \frac{1}{\frac{1}{2}(e^x - e^{-x})} = \frac{1}{\sinh(x)} = \frac{1}{\sqrt{\cos^2 h(x) - 1}} = \frac{1}{\sqrt{y^2 - 1}}$$

Nun Variablen tauschen

$$y = \frac{1}{\sqrt{x^2 - 1)}}$$

Gegeben: f(x) = y = tanh(x)
Gesucht: arctanh(x)

$$f(x) = y = \tanh(x) = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$$

$$f'(x) = y` = 1 - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2} = 1 - \tanh^2(x)$$

$$\text{g'(y)} = \frac{1}{f'(x)} = \frac{1}{1 - \tanh^2(x)} = \frac{1}{1 - y^2}$$

Nun Variablen tauschen

$$y = \frac{1}{1 - x^2}$$

Gegeben: f(x) = y = coth(x)
Gesucht: arccoth(x)

$$f(x) = y = \coth(x) = \frac{(e^x + e^{-x})}{(e^x - e^{-x})}$$

$$f'(x) = y` = 1 - \frac{(e^x + e^{-x})^2}{(e^x - e^{-x})^2} = 1 - \coth^2(x)$$

$$\text{g'(y)} = \frac{1}{f'(x)} = \frac{1}{1 - \coth^2(x)} = \frac{1}{1 - y^2}$$

Nun Variablen tauschen

$$y = \frac{1}{1 - x^2}$$